PIPELINE ENGINEERING

ALTERNATIVE PLANNING MODELS

With the advance of mixed integer linear and non-linear programming methods, the optimization and planning of systems became more systematic. When intensive computations became viable, these planning models were extended to deal with data uncertainty through the so-called Two stage stochastic programming. We start presenting an ultra-simplified planning model, which should serve as basis for more complex and detailed models. In a later Module, design and optimization under uncertainty as well as risk management using these models are covered.

We consider the case of a varying demand increasing in the future and we assume such demand will be covered. The time is discretized in several time periods (usually years) and what is needed to be determined is

- Initial pipeline Capacity. It can be the demand in the first few periods, or the demand at the end of all periods. In the former case, expansions will be needed, while in the latter no expansions are needed and the pipeline will operate under-capacity for a period of time.
- Expansions are to be achieved in some future period (to be determined) by adding a compressor at the middle point of the pipeline. This can be generalized to many compressors at many times and locations.

Simplified Capacity Expansion Model

SETS

T: Time periods $,t = 1,\dots,NT$

K: Compressor to be added/expanded. Compressors in parallel to be added to existing ones.

VARIABLES

Y_{kt}: *Expansion in period t takes place (Y_t=1), does not take place (Y_t=0)*

E_{kt}: *Expansion of capacity in period t. Addition of compressors*

Q_t: *Total pipeline capacity in period t.*

W_t: *Utilized capacity in period t.*

PARAMETERS

sp_t : *Sales price in period t*

c_t : *Cost coefficient in period t (gas + operating costs).*

α_t : *Variable cost of expansion in period t*

β_t : *Fixed cost of expansion in period t*

L_t *: Discount factor for period t*

D_t *: Demand of gas for period t*

E_{kt}^L, E_{kt}^U *: Lower and upper bounds on expansion in period t*

$CI_t^{\max}$ *: Maximum capital available in period t*

$NEXP$ *: Maximum number of expansions*

MODEL EQUATIONS

The objective function maximizes the net present value of the project. We try this linear objective first, although it is known that sometimes return of investment (ROI) is a better profitability measure. Moreover, under conditions where one can choose not to invest in unprofitable parts of the project, maximizing NPV is not equivalent to maximizing ROI and leads to different capital investments.

The net present value is calculated as the sum of present values of revenues ($(ps_t W_t)$ minus operating costs $(c_t W_t)$ minus the amount that is invested. $(\alpha_t E_t + \beta_t Y_t)$.. This investment is composed of a fixed part $(\beta_t Y_t)$, which is only counted when $Y_t = 1$, and a part that is proportional to the size of the expansion. Note that the model considers the first construction and compressor installation as an "expansion" in period 1. The fixed and variable costs include the pipeline and the compressor. True expansions at a later time have fixed and variable costs that correspond to extra compressors.

Finally, one needs to note that while *NPV* is straightforward to calculate, *ROI* based on various investments on the same project are difficult to handle, especially if only one measure is sought.

$$Max\ \ NPV = \sum_{t \in T} L_t \left\{ (ps_t - c_t) W_t - \sum_{k \in K} (\alpha_t E_{kt} + \beta_t Y_{kt}) \right\} \qquad (1\text{-}1)$$

Subject to (s.t.)

The following constraint controls the minimum and maximu size of an expansion. Note that this constraint forces E_t to be zero when $Y_t=0$.

$$Y_{kt} E_{kt}^L \le E_{kt} \le Y_{kt} E_{kt}^U \qquad t \in T \qquad (1\text{-}2)$$

The following constraint updates the pipeline capacity by adding the expansions to existing capacity .

$$Q_t = Q_{t-1} + \sum_{k \in K} E_{kt} \qquad t \in T$$

(1-3)

The number of expansions is controlled

$$\sum_{t\in T}\sum_{k\in K} Y_{kt} \le NEXP \qquad (1\text{-}4)$$

Investments are subject to pre-established maximums

$$\sum_{k\in K}\left(\alpha_t E_{kt} + \beta_t Y_{kt}\right) \le CI_t^{\max} \qquad t \in T \qquad (1\text{-}5)$$

Current flow is lower than capacity

$$W_t \le Q_t \qquad t \in T \qquad (1\text{-}6)$$

Current flow is equal to the demand. As an aside, one can handle this issue through an inequality ($W_t \le D_t$), with the difference subtracted as a financial penalty in the objective function.

$$W_t = D_t \qquad t \in T \qquad (1\text{-}7)$$

Binary and continuous variables are declared.

$$Y_{kt} \in \{0,1\} \qquad t \in T \qquad (1\text{-}8)$$

$$E_{kt}, Q_t, W_t \ge 0 \qquad t \in T \qquad (1\text{-}9)$$

These are the deficiencies of the above simplified model.

a) Pipeline diameter(s) is (are) pre-selected.
b) Simulation is needed to establish the final expanded capacity.
c) Pressures and compressor work is not computed.
d) Costs are the same for all locations.
e) Investments in the future are decided beforehand and discounted at the common rate. Investors would rather put the money in more lucrative enterprises than use banking tye of interest.

The following expanded model considers the choice of diameter (s), calculates pressures, work, and adds budgeting so that reinvestments of proceeds is considered. To do all this, a new set of pipeline segments is added. This is commented out through the model

Detailed Capacity Expansion Model

SETS

T: Time periods $,t = 1,\ldots,NT$

K: Compressor to be added/expanded. Compressors in parallel to be added to existing ones.

I: pipeline segments

End(I)⊂I: pipeline segments connected to delivery points.

VARIABLES

Q_{it}*: Pipeline flow for section i in period t.*

QC_{kt} *: Flowrate for compressor k in period t*

H_k*: Power of Compressor k.*

HC_k*: Installed power capacity of compressor k.*

Y_{kt}*: Compressor k is installed in period t*

NR_t*: Net revenue in period t*

CI_t*: Capital investment (from investors) in period t*

C_t*: Cash at the end of period t*

D_i*: Diameter of pipeline segment i*

P_{it}*: Pressure at time t at the end of segment i*

Pb_{it}*: Pressure at time t at the beginning of segment i*

PARAMETERS

c_t *: Cost coefficient in period t (gas + operating costs).*

ps_t : price of gas. Includes gas needed to run compressors

pe_t *: penalty paid in period t for not meeting demand*

$Disc_t$ *: Discount factor for period t*

L_i*: Length of segment i.*

Δz_i *: Elevation change of segment i.*

Dem_{it} *: Demand of gas for period t at the end of segment i.*

$C_t^{\max}$ *: Maximum capital available in period t*

PC_{kt} *: discharge pressure compressor power equation*

P_k^{in} *: Inlet pressure of compressor k*

$Hinst_k$: *Installed capacity of compressor k*

n: *Compressor power equation coefficient .*

α_{kt} *: Variable cost of compressor k in period t*

β_{kt} *: Fixed cost of compressor k in period t*

α_i^P : *Variable cost of section i in period t*

β^P : Fixed cost associated to pipeline construction.

ψ_{si} *: fraction of flow from segment s that feeds into segment i*

$\vartheta_{ki} = 1$ *if compressor k feeds section i.*

$\delta_{it}^{(1)}, \delta_{it}^{(2)}$: constants for flow vs. pressure equation

λ_{si} *=1, if segment s feeds into segment i in a mixer node*

σ_{si} *=1, if segment s feeds into segment i in a splitter node node*

φ_k *: coefficient of compressor power equation*

η_k *: efficiency of compressor k*

MODEL EQUATIONS

$$Max\ \ NPV = \sum_{t \in T} Disc_t \left\{ NR_t - CI_t \right\} \qquad (1\text{-}10)$$

s.t.

We introduce a new variable to account for the extra cash added.

$$CI_t \le C_t^{\max} \qquad t \in T \qquad (1\text{-}11)$$

We allow delivering less gas than the demand and pay a penalty.

$$Q_{it} \le Dem_{it} \qquad t \in T \qquad i \in End(I) \qquad (1\text{-}12)$$

For cases where more than one entry point to the pipeline is considered and when there is branching, material balances are needed at each segment:

$$Q_{it} = \sum_{s \in S} \psi_{si} Q_{st} + \sum_{k \in K} \vartheta_{ki} QC_{kt} \qquad t \in T \qquad (1\text{-}13)$$

where ψ_{si} =1 is the fraction of flow from segment *s* that feeds into segment *i* and $\vartheta_{ki} = 1$ if compressor *k* feed segment *i.* This equation can be omitted and the flowrates can be considered parameters in the case where is only one supply compressor at the pipeline start, or the expansion flows for each compressor are decided beforehand. Next, we compute the pressures in each segment as follows:

$$Q_{it}^2 = \frac{\delta_{it}^{(1)} D_i^5}{L_i}(P_{it}^2 - Pb_{it}^2) + \frac{\delta_{it}^{(2)} \Delta z_i D_i^5}{L_i} \qquad t \in T \qquad (1\text{-}14)$$

where Pb_{it} is the initial pressure in the segment. Thus for connectivity we write

$$P_{it} = \sum_{s \in S} \lambda_{si} Pb_{st} \qquad t \in T \qquad (1\text{-}15)$$

where λ_{si} =1, if segment s feeds into segment i in a mixer node, and

$$Pb_{it} = \sum_{s \in S} \sigma_{si} P_{st} \qquad t \in T \qquad (1\text{-}16)$$

where σ_{si} =1, if segment s feeds into segment i in a splitter node node, and

The power utilized in each expansion compressor is related to the compressor actual flow QC_k. Note that we consider the efficiency constant, but one could make it pressure dependent.

$$H_{kt} = \frac{\varphi_k}{\eta_k}\left[\left(\frac{PC_k}{P_k^{in}}\right)^{\frac{n-1}{n}} - 1\right] QC_k \qquad t \in T \qquad (1\text{-}17)$$

The following equation assigns the outlet pressure of the compressor to a pressure of some segment.

$$PC_{kt} \geq \sum_{i \in I} \vartheta_{ik} P_{it} \qquad t \in T \qquad (1\text{-}18)$$

where ϑ_{ik} is a zero-one variable assigning a compressor to a node. Sometimes PC_{kt} is considered to be a parameter, hence the inequality.

The following equation limits the actual power to be smaller than the installed power

$$H_{kt} \leq HC_{kt} \qquad t \in T \qquad (1\text{-}19)$$

Because HC_{kt} is the same for all periods after the compressor installation, one needs a variable that will account for the installed power once ($Hinst_{kt}$). This variable will be used in the cost equation.

$$Hinst_{kt} \geq HC_{kt} - (1 - Y_{kt})\Omega \qquad t \in T \qquad (1\text{-}20)$$

$$Hinst_{kt} \leq Y_{kt}\Omega \qquad t \in T \qquad (1\text{-}21)$$

$$\sum_{t\in T} Y_{kt} \le 1 \qquad t \in T \qquad (1\text{-}22)$$

When $Y_{kt} = 1$, which can only happen once, $Hinst_{kt}$ is limited to be larger than HC_{kt} and smaller than an upper bound Ω. Conversely, when $Y_{kt} = 0$, $Hinst_{kt}$ is equal to zero. Finally, we link HC_{kt} to $Hinst_{kt}$

$$HC_{kt} = HC_{k(t-1)} + Hinst_{kt} \qquad t \in T \qquad (1\text{-}23)$$

Budgeting: We consider a certain cash (C_t) (usually kept in the bank) and not taken out to be given back to the investors. What is given back to the investors is accounted for by the variable NR_t. The budgeting constraint includes a cash balance, where investments and revenues from sales are added to the previous period, while money spend in operations as well as penalty for not meeting demand, money spent on expansions and proceeds given back to investors. Note that unlike the previous model the investments and proceeds are made at the end of the period, and the revenues and costs are computed at the end. First period is construction only.

$$C_1 = CI_1 - \sum_{k\in K} \left(\alpha_{k1} Hinst_{k1} + \beta_{k1} Y_{k1}\right) - \sum_{i\in I} \alpha_i^P D_i - \beta^P \qquad (1\text{-}24)$$

$$\begin{aligned} C_t = C_{t-1} + CI_t + ps_{t-1} \sum_{i\in End(I)} Q_{i(t-1)} - pe_{t-1} \sum_{i\in End(I)} \left(Dem_{i(t-1)} - Q_{i(t-1)}\right) & \\ - \sum_{k\in K;t\in T} \left(\alpha_{kt} Hinst_{kt} + \beta_{kt} Y_{kt}\right) - NR_t & \qquad t \in T\,;\, t \notin t_1 \qquad (1\text{-}25) \end{aligned}$$

$$Q_{it}, C_t, HC_{kt}, H_{kt}, D_i, NR_t, CI_t, PC_{kt}, P_{it}, Pb_{it} \ge 0 \qquad k \in K\,;\; i \in I\,;\; t \in T \qquad (1\text{-}26)$$

We note now that ALL equations relating flow to pressures can be substituted by equations relating the initial pressure of a source point of a path p through the network like follows:

$$P_{in(p)t}^2 - P_{end(p)t}^2 \ge \sum_{r\in p} \left[Q_{rt}^2 \frac{L_r}{\delta_{rt}^{(1)} D_r^5} - \frac{\delta_{rt}^{(2)} \Delta z_r}{\delta_{rt}^{(1)}} \right] \qquad t \in T\,;\, p \in P \qquad (1\text{-}27)$$

These paths connect supply and delivery points as well as compressor points. Additional tricks to make the model more easily solvable are available and will be discussed by the instructor.